Signals & Systems
Student ID Number: ________________________________

Unified Engineering — Fall 2007
Quiz 5

Tuesday December 18

S&S Total: 70/120 points
Question 1: 35/70 points
Question 2: 35/70 points

Question 1 (35 points)

An LTI system is described by the following state-space matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -4 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Note that the system has two outputs, $y_1(t) = x_1(t)$ and $y_2(t) = x_2(t)$. The state transition matrix for the system is

$$\Phi(t) = \begin{bmatrix} \cos 2t & \frac{1}{2}\sin 2t \\ -2\sin 2t & \cos 2t \end{bmatrix}.$$

S1(a) [15 pt] Compute the output response of the system to a unit step input, $u(t) = 1$, $t > 0$. The initial state of the system is $\vec{x}(0) = [1 \;\; 0]^T$.

S1(b) [10 pt] Sketch your response from S1(a) on two separate plots: $y_1(t)$ versus t and $y_2(t)$ versus t. On each plot indicate clearly the values of the magnitude of oscillations and the period of oscillations.

S1(c) [10 pt] Create a trajectory plot of the system response by plotting y_2 versus y_1. Indicate clearly on the plot the values of the important characteristics of the trajectory, such as the central point and the minimum/maximum values.

Signals & Systems Question S1
Student ID Number: ______________________________

Signals & Systems Question S1
Student ID Number: ______________________________

Student ID Number: ______________________________

Question 2 (35 points)

An aircraft has roll angle $\phi(t)$ and roll rate $\dot{\phi}(t)$. The roll dynamics satisfy the following second-order differential equation

$$\ddot{\phi} = -c\dot{\phi} + u, \tag{1}$$

where

$$\begin{aligned} c &= \text{aerodynamic damping coefficient,} \\ u &= \text{input command to the aircraft aileron system.} \end{aligned}$$

The heading angle of the aircraft velocity vector, $\Psi(t)$, satisfies the following differential equation

$$\dot{\Psi} = \frac{a_l}{V}, \tag{2}$$

where the aircraft velocity, V, is assumed to be constant. a_l is the lateral acceleration in the horizontal plane, which is produced by the roll angle according to the equation

$$a_l = a_L \sin\phi, \tag{3}$$

where a_L is the acceleration due to lift.

Use the following parameter values:

$$\begin{aligned} c &= 1 \text{ rad/s} \\ a_L &= 9.8 \text{ m/s}^2 \\ V &= 98 \text{ m/s.} \end{aligned}$$

S2(a) [15 pt] Create a linear state-space model with the aileron command, $u(t)$, as input, and the heading angle, $\Psi(t)$, as the output. You can assume that the roll angle is always small.

S2(b) [20 pt] Design a full-state feedback controller that places the eigenvalues of the controlled system at $s = -1$ and $s = -1 \pm j$.

Signals & Systems Question S2
Student ID Number: ____________________________________

Signals & Systems Question S2
Student ID Number: ______________________________